Quandaries of Forest Area, Volume, Biomass and Carbon Explored with the Forest Identity

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SUMMARY

The Forest Identity defines the four valued forest attributes of area, growing stock volume, biomass and sequestered carbon in terms of the four measurable variables of area, density of volume per hectare, an allometric ratio, and the carbon concentration in biomass. A single, synoptic chart maps the rise and fall of the variables and attributes of the Identity and graphically separates nations with improving versus deteriorating forest attributes. Exercises examine rates of change in 50 nations with the greatest reported growing stock volume, in European and African nations around the Mediterranean and in Central American and Caribbean lands. Other exercises examine national variations cloaked by global sums and transparently calculate carbon sequestered in nations’ forests. Final exercises examine the correlation between timber harvest and deforestation and the related advantage of harvesting fast growing, warm forests or plantations. The frequency of expanding forests and the advantage of harvesting fast growing forests sustains hope that deforestation accompanied by deteriorating attributes is not inevitable.

INTRODUCTION

People value forests as solitude far from crowds, as lumber for building homes, as fuel for warmth and energy, and as carbon orchards withholding climate-changing greenhouse gas. Three valuable forest attributes are their expanse, their biomass, and their sequestered carbon. A fourth valued attribute is their volume of timber large enough to harvest profitably, which foresters call growing stock volume and we shall simply abbreviate as volume. Until glibly listed attributes are defined in measurable quantities, our knowledge remains meager and unsatisfactory (Thomson, 1883). Accordingly with the Forest Identity, Kauppi et al. (2006) defined the attributes listed in Table 1 by the measurable variables also listed in the Table.

The brevity of the introduction of the Identity by Kauppi et al. requires amplifying its derivation and also examining more forest phenomena with it. The synoptic chart that displays gains and loss of volume in terms of the variables of area and density needs extension to biomass and to carbon, recently made important by fears of climate change and anticipation of producing biofuel.

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THE DATA AND METHOD

The UN’s Food and Agriculture Organization (FAO) publication of the Forest Resource Assessment in 2005 brings up to date the series begun in 1946. We have cited this global appraisal as Food and Agriculture Organization (2005) in the references and shall call it FRA2005. Mather (2005)
described the evolution of the Forest Resource Assessments and their quality. Appraising the dynamic rather than static state of forests demands consistent surveys. Fortunately, FRA2005 adjusted 1990 values for subsequently improved methods, making them consistent with 2005 values. Thus we could calculate rates of change in nations as the average annual rates during the 15-years, 1990 to 2005. For the slow rates of annual change encountered, changes of natural logarithms (ln) and percentage changes are practically interchangeable.

\[
\ln \left( \frac{\text{value in 2005}}{\text{value in 1990}} \right)/15 \approx \text{average percentage change per year.}
\]

Despite the improvement represented by FRA2005, only 214 countries reported forest areas and 144 reported volumes in 1990 and 2005. Others must be omitted from our analyses. For example, Australia and Germany did not report areas for the two years.

U.S. reports of timberland from 1953 to 2002 provided data for analyses among regions, with more than one time span, and with the consistent definitions possible in a single nation (Smith et al., 2004). The U. S. A. defines timberland as accessible, legally harvestable and capable of producing 1.4 m³/ha/yr.

Kauppi et al. (2006) introduced the analytical method of the Forest Identity compactly. In the section Definition and explication of the Forest Identity below, we derive it thoroughly.

**DEFINITION AND EXPLICATION OF THE FOREST IDENTITY**

**AREA OF FOREST AND VOLUME OF TIMBER**

The wider a forest, the more wilderness solitude it provides. A wider forest provides more attractive landscape, habitat for biodiversity, and more watersheds to collect water. Begin the Identity with the attribute of area \( A \) hectares (ha). In the International System of Units (SI), 1 hectare equals \( 10^4 \) m² or 2.47 acres.

Forest area \( A \) dominates consideration of deforestation. For example, the key findings of FRA2005 concern the total hectares of forest lost. In another example of concentration on area, the Millennium Ecosystem Assessment stated, “The global area of naturally regenerating forest has declined throughout human history and has halved over the past three centuries” (Hassan, Scholes, Ash, 2005). Pooling markedly different hectares in a global area, however, cloaks illuminating differences. For example, at 350 m²/ha an average forest hectare of Switzerland or French Guiana holds more than 30 times the volume on a hectare of forest in Niger, Saudi Arabia, or Uzbekistan. Concentrating on area alone is like gambling without noticing the different colors of the chips. Fortunately the volumes per hectare reported by FRA2005 provide weights for the logical combination of differing hectares.

Forest science might have proceeded logically from photosynthesis to biomass and finally reached the portion for profitable harvest. Had that been the history, inventories of biomass would be on hand in tons per hectare. In SI units, 1 ton or tonne equals \( 10^6 \) grams or 1.10 short tons of 2,000 pounds. If the logical order photosynthesis-to-biomass-to-timber were the history of forest measurement, the second variable in the Identity would be the biomass of accumulated photosynthetic product. Then when practical people wanted to know how much lumber they could harvest, they would proceed from photosynthesis and biomass on to volume. They would multiply biomass by an allometric ratio of volume to biomass. (See allometry below.) But for a practical assessment of the valuable timber products, especially lumber, in a forest, first adding up photosynthesis or weighing the biomass of all the branches, twigs and leaves would have been a diversion. So foresters went to the field and measured the practical attribute of volume, directly. Because foresters’ measurements of volumes, not biomass inventories are on hand, volume follows area in the Identity. When biomass is to be calculated, it is translated from volume rather than the other way around.

Recall that here volume abbreviates the forester’s longer phrase growing stock volume. The state of volume \( V \) m³ in a forest of area \( A \) ha is identical to and defined by its area multiplied by its density \( D \) m³/ha.

\[
V = A \times D
\]

\[
\ln(V) = \ln(A) + \ln(D)
\]

Rates of change from 1995 to 2005 rather than static states \( V, A, \) and \( D \) introduce the dimension of time and a dynamic view.

\[
d\ln(V)/dt = d\ln(A)/dt + d\ln(D)/dt
\]

An alternative expression of the derivatives in the preceding equation emphasizes that the rates are relative ones.

\[
dV/dt / V = dA/dt / A + dD/dt / D, \text{ per year}
\]

For the slow rates actually encountered, percentage changes per year closely approximate the logarithmic changes per year. For example, the product of a 3% change in \( A \) and 4%
change in $D$ is a 7.1% change in $V$. The corresponding changes in logarithms are .0296, .0392 and .0688. Letting lower case letters be annual percentage changes of the variables $a$ and $d$ leads to an identity for the changing attribute of national volume $v$.

$$v = a + d, \text{ percent per year.}$$

That is, volume grows annually by a percentage approximately equal to the rate of changing area for wilderness, watershed, or managed forest plus the rate of changing density of timber for products.

The areal extension $a$ and growing density $d$ for nations can be plotted as longitude and latitude on a single chart (Figure 1). Calling left and right on the chart longitude and up and down latitude avoids any implication that $d$ is a function of $a$. The nations with expanding forests lie east or right, and those with shrinking forests lie west or left of the zero meridian at $a = 0$. Similarly, nations whose forests grew denser lie north or above the equator of unchanging density at $d = 0$.

The boundary marked by the red line on the chart, representing $d = -a$, separates nations with increasing from nations with decreasing national volumes. Nations northeast or above the red line gained, and those southwest of the line are losing volume. In any nation on the red line, the rates of change of forest area and density exactly counter one another, making its $v = 0$. The forests in the nation represented by the point $(a,d)$ on Figure 1, of course, are expanding at $a$ per year and growing denser at $d$ per year. The horizontal distance westward on the chart from point $(a,d)$ to the point $(-d,d)$ on the line, $a = -d$ equals, the change of volume per year,

$$v = [a - (-d)] = [a + d]$$

The vertical distance north to south on the chart from $(a,d)$ down to the point $(a,-a)$ on the line $a = -d$ also equals $v$.

Nations in quadrant I of Figure 1 are gaining volume because both area and density are increasing. Nations above or northeast of the red line for $a = -d$ in quadrant II are gaining volume because their forests are growing density faster than their areas are shrinking. Nations above the red line in quadrant IV also are gaining volume because their forests are expanding faster than their densities are falling. Similar reasoning means that all nations southwest of the red line for $a = -d$ lose volume, those in quadrant III because both area and density fall.

The red boundary separates nations above or northeast whose changed area $a$ and density $d$ increase their volume $v$ from those nations below or southwest of the line whose changed area and density cut their volume. Thus the global kaleidoscope of nations’ changing area, density and volume, too, can be represented on one chart of a single plane with the dimensions of $a$ and $d$. Because the representation comprises a general view of the whole dynamic database about the variables of area and density and the attribute of volume around the globe during a single time span, we call the representation a synoptic chart.

**ACCUMULATED BIOMASS**

As interest grew in forest ecosystems, it grew in the biomass that energizes them. And, as interest grows in forests as practical biofuel, it grows in biomass, which encompasses more than the volume of trees big enough to cut for lumber. The forest variables $a$ and $d$ plus the attribute $v$ represented on Figure 1 may satisfy seekers after solitude and protectors of watersheds, and lumbermen, too, but the chart has not yet encompassed biomass. The biomass above ground includes stumps, branches, bark, seeds and foliage in addition to the volume $V$ of stems large enough for commercial timber.
Fortunately, estimates of biomass need not wait on the harvest of all small stems, branches and so forth. Instead, foresters’ accumulated surveys of \( V \) (m\(^3\)) timber big enough to cut into timber can be converted into the \( M \) (tons) biomass. A complete inventory of biomass would include roots and the organic matter in the soil. For now, however, we define \( M \) as the above ground organic matter in all living biomass above the soil.

An allometric relation in the form of the ratio \( B \) (tons of biomass per m\(^3\) of volume) connects biomass to timber. Authors have called \( B \) the Biological Expansion Factor, BEF (Schroeder et al., 1997; Brown, Schroeder, 1999; Brown, 2002). Allometry is the study of the changing proportion of various parts of an organism during growth. Of allometry, D’Arcy Thompson (1917) wrote, “The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.”

Forests of small trees have some biomass but few or no trees large enough to yield lumber. As trees grow, their trunks become a larger and larger portion of forest biomass, lowering the \( B \) (tons of biomass per m\(^3\) of volume). So,

\[
M \text{ (tons)} = V \text{ (m}^3\text{)} \times B \text{ (tons/m}^3\text{)} = A \text{ (ha)} \times D \text{ (m}^3\text{/ha)} \times B \text{ (tons/m}^3\text{)}
\]

The tons and m\(^3\) dimensions of the allometric variable \( B \) reflects its dual function. \( B \) encompasses the specific gravity \( \rho \) (tons/m\(^3\)) of wood to translate volume into wood. \( B \) also incorporates the dimensionless fraction \( f \) (tons of biomass per ton of wood in the volume \( V \)). Thus, \( B = \rho / f \). Also, the fraction \( f \) of biomass in volume \( V \) equals \( \rho / B \). At the improbable limit of all biomass becoming wood in the volume \( V, f \) becomes 1, and \( B \) equals \( \rho \). Because the specific gravity of much timber is about one half (Birdsey, 1992), the lower limit of \( B \) is about one half. Agronomists call the ratio of grain to biomass, the harvest index, and have found that its rise from about a third to a half explains much of the wheat yield increases during the 20\(^{th}\) century (Evans, 1993).

Visualize an idealized forest where, from the beginning at time 0, small trees grow along the straight green line in Figure 2. During each time unit, they grow 1 unit of \( M/A \) biomass per area. Later, after time 1, trees grow large enough to be harvested, causing density \( D \) then to grow along the straight blue line at 1 unit per unit time. The linear growths of \( M/A \) tons/ha and \( D \) m\(^3\)/ha cause the fraction \( f \) in timber to rise along the curvilinear red line. The hypothetical \( B \) biomass per volume falls curvilinearly along the black line. Measured \( B \) frequently declines from 5 to 1 tons of biomass per m\(^3\) of timber (Brown, 2002). Because \( f \) equals \( \rho / B \) and \( \rho \) equals 0.5, these allometric ratios from 5 to 1 represent volume holding a tenth to half of the aboveground biomass. Although a real rather than idealized forest would grow along S-shaped curves rather than straight lines, the simple straight lines of Figure 2 nevertheless demonstrate how the curves of \( B \) and \( f \) get their shapes.

![Figure 2. An idealized forest where the biomass per area \( M/A \) grows 1 ton/ha per time from time 0, whereas timber density \( D \) grows at 2 m\(^3\)/ha, but after time 1. The ratio \( f \) represents (tons of timber per ton of biomass), and over time it approaches 1. As the trees grow, \( B \) (tons of biomass per m\(^3\) of volume) approaches the specific density \( \rho \) of wood, near 0.5.](image3.png)

Conveniently for adding the changing biomass \( m \) to area \( a \), density \( d \) and volume \( v \) on a single synoptic chart, in many forests the ratio \( B \) declines fairly regularly as \( D \) increases. In a representative example, Brown and Schroeder (1999) reported for hardwood and spruce-fir forests in the U. S. A.,

\[
\ln(B) = \beta_0 - \beta_1 \times \ln(D) = 1.9 - 0.34 \ln(D)
\]

Exceptions to the general decline of \( B \) with \( D \) do exist. In pine forests, they found no regular relation between \( B \) and \( D \). Tropical forests generally have more biomass per volume and thus higher \( B \) than temperate forests. Fortunately again, the regression coefficient \( \beta_1 \) connecting changing biomass to changing area and density in the synoptic chart varies less than \( B \) itself. Fang (2006) found that in China, India, and Japan, \( \beta_1 \) varied only from 0.28 to 0.36. In Europe and Russia, \( \beta_1 \) was less than 0.1. For now, let \( \beta_1 \) be 0.3, decreasing \( B \) by 3% when \( D \) rises by 10%. Then when

\[
\ln(B) = \beta_0 - 0.3 \ln(D)
\]
\[ M (\text{tons}) = A (\text{ha}) \times D (\text{m}^3/\text{ha}) \times B (\text{tons}/\text{m}^3), \]
\[ \ln(M) = \ln(A) + \ln(D) + \ln(B) = \ln(A) + \ln(D) + \lbrack \beta_0 - 0.3 \ln(D) \rbrack, \]
and
\[ \frac{dV}{dt}/V = \frac{dA}{dt}/A + \frac{dD}{dt}/D + (-0.3) \frac{dD}{dt}/D \]

Letting lower case letters be annual percentage changes leads to an identity for a national change in biomass \( m \):
\[ m = a + d + b = a + d + \beta_1 d = a + (1 - 0.3) d = a + 0.7 d, \]
percent per year.

Increasing area \( a \) by 1% increases \( m \) by 1%. Because \( B \) decreases as \( D \) increases, however, increasing \( d \) by 1% increases \( m \) by only 0.7%. Although area and density have the same leverage for increasing volume, area has more leverage than density for increasing biomass.

Separating nations with increased from ones with decreased biomass therefore requires the green line \( d = -a/0.7 \) on Figure 3. When \( B \) decreases 3% as \( D \) rises 10%, the rate of change of area, timber density and the ratio \( B \) of forests in any nation on the green line exactly counter one another. That is, on the green line, \( m \) equals 0. For example, the forests in the nation represented by the point \((a,d)\) on Figures 1 and 3 expanded at \( a \) per year and grew denser at \( d \) per year. The horizontal distance on the chart from point \((a,d)\) to the point \((-0.7 \, d, \, d)\) on the green line of Figure 3 is the annual change of biomass,
\[ m = [a - (-0.7 \times d)] = [a + 0.7 \times d]. \]

Because \( B \) falls as \( D \) rises, biomass grows 0.3 \( d \) more slowly than \( v \) in quadrant II and falls 0.3 \( d \) more slowly than \( v \) in quadrant IV. The decrease of \( B \) with increasing \( D \) retards the rise of \( m \) in quadrant II and its fall in quadrant IV. The horizontal distance between the green and red lines represents the retardation 0.3 \( d \). The vertical distance \([d - (-a/0.7)]\) from \((a,d)\) to the point \((a,-a/0.7)\) on the green line equals \(m/0.7\) change of biomass.

If a nation gained volume but lost biomass, it would appear between the red and green lines in quadrant II. Or, if it lost volume but gained biomass, it would appear between the red and green lines in quadrant IV. For example, a nation annually losing 1% area and growing density at 1.1% would gain volume but lose biomass. Alternatively, a nation annually gaining 1% area and losing 1.1% density would lose volume but gain biomass.

We return to the subject of roots that an earlier section postponed when \( M \) was defined as the above ground organic matter. Fortunately, a correlation between root and shoot biomass allows an interpretation of the green line for unchanging \( M \) and \( m = 0 \) in terms of the sum of root plus shoot biomass. Roots increase nearly linearly as one fourth of shoot mass (Mokany, Raison, Prokushkin 2006). Accordingly, the biomass of the total roots plus shoots equals a nearly constant 1.25 times \( M \), which changes at the relative rate \( m \). The constancy of the 1.25 means that the green line separating nations gaining from those losing aboveground biomass also separates those gaining and losing all biomass, above and below ground.

Nations in quadrant I of Figure 3 gain biomass because both area and density increase. In nations above or northeast of the green line in quadrant II, increasing density compensates for lost area despite a decline in \( B \) with greater \( D \). Nations above the green line in quadrant IV also gain biomass because 1) their areas expand and 2) a decline in density raises the ratio \( B \) of biomass to volume. Similar reasoning leaves all nations southwest of the green line losing biomass.

**Figure 3.** A synoptic chart separating nations with increasing biomass northeast of the green line from those with decreasing biomass southwest of the green line. The annual change \( a \% \) of area is longitude, and the annual change \( d \% \) of density is latitude. The point \((a, d)\) is marked in quadrant I. Point \((-0.7 \, d, \, d)\) in quadrant II with longitude equal to \(-0.7 \, d \) and the point in the quadrant IV with latitude equal to \(-a/0.7 \) lie on and define the diagonal green line where biomass does not change, and \( m \) equals zero. If the biomass above ground is a constant multiple of below ground biomass, the sum above and below does not change on the green line. If the carbon concentration in biomass does not change, the green line separating nations with increasing from those with decreasing biomass becomes the separation between rising and falling carbon sequestration, too. As in Figure 2, the red line separates nations with increasing from those with decreasing volume \( v \).
those in quadrant III because both area and density fall. The allometric relation between the variables $B$ and $D$ permits a two-dimensional synoptic chart of changes of the attributes $A$, $D$, $V$ and $M$ in forests.

**CARBON ORCHARDS**

The fear that more carbon dioxide in the atmosphere will change the climate has focused attention upon the carbon captured in forests by photosynthesis. If both natural and planted forests perform as carbon orchards, they diminish the atmospheric burden of carbon dioxide. Estimating $Q$ tons of carbon sequestered aboveground in the forest requires specifying the concentration $C$ tons carbon per ton biomass. The Forest Identity integrates four variables into the attribute $Q$.

$$Q \text{ (tons carbon)} = A \text{ (ha)} \times D \text{ (m}^3/\text{ha)} \times B \text{ (tons/m}^3) \times C \text{ (tons carbon/ton biomass)}.$$  

After converting the variables to logarithms, differentiating the logarithms, and replacing the derivatives with annual percentage changes, the percentage change in carbon $q$ becomes the sum of four changes.

$$q = a + d + b + c,$$

.percent per year.

Since $C$ ranges only from 0.50 to 0.53 (Birdsey, 1992), let $C$ be constantly 0.5, and so $c = 0$. Although $c$ may equal 0, $C$ must remain in the Identity to maintain correct dimensions. If $c = 0$, the green line separating nations with increasing biomass from those with decreasing biomass becomes the separation from falling carbon sequestration, too.

Values for the U.S.A. in 2005 reported by FRA2005 illustrate the Identity:

$$Q \text{ tons} = A \times D \times B \times C$$

15,826 million tons = $303,089 \times 116 \times 0.9 \times 0.5$ tons/ton

If $a$ and $d$ changed as FRA2005 reports for 1990 to 2005, if $B$ falls 3% for each 10% rise of $D$, and if $C$ is constant, the carbon sequestered in U. S. forests increased 0.45% annually.

$$q = a + d + b + c$$

0.45 = 0.10 + 0.49 - 0.14 + 0, percent per year.

Estimates of the variables $a$, $d$, $b$ and $c$ transparently define the attribute $q$ by measured variables. If all biomass equals a constant times that above ground, the same variables that define the changing carbon above ground define the change in all forest carbon. Equally, an estimate of the changing attribute $q$ imposes the discipline of specifying the changing variables $a$, $d$, $b$ and $c$ that could cause it. Although uncertainties in the allometric ratio $B$ may affect the location of the green line, the red line and the points for each nation are the immutable result of FRA2005 reports and of the logic that volume equals area times density. The values reported in FRA2005 are, of course, subject to error, but because the Forest Identity is an identity, the variables $A$, $D$ and $V$ add no further errors. Arranged in the Identity, a negligible variation in carbon concentration $C$ plus an allometric ratio $B$ that varies with density $D$ make a synoptic view of changing area and timber density into a single, synoptic chart for carbon as well as volume and biomass, above and below ground.

**QUANDARY I: CAN MANY DATA ABOUT REAL FORESTS BE VISUALIZED ON A SINGLE SYNOPTIC CHART?**

Concrete data impart reality to barren Figure 3. Among the nations tabulated in FRA2005, the 50 with the most growing stock in 2005 had 80% of all reported forest area and 95% of all reported volume. Although only part of the FRA2005 data, these 200 numbers from 50 quartets of area and density in two years nevertheless boggle the mind. Happily the synoptic chart in Figure 4, built on the Identity, renders the bewildering 200 values of the variables plus their interpretation as four attributes into a single, comprehensible view. As in introductory Figures 1 and 3 above, the change $a$ in area increases rightward from west to east in Figure 4, while the change of density $d$ increases upward from south to north.

As proven above, national rates $v$ of changing volume equal the distances straight east or west, right or left, from the diagonal red line representing $a = -d$. National rates $m$ of changing biomass and $q$ of carbon equal the distances straight east or west from the diagonal green line representing $a = -0.7 \times d$. Although nations that lose volume will generally lose biomass, exceptions could arise. Nevertheless, the synoptic chart of these 50 forested nations shows none between the green and red lines indicating changes of volume and biomass in different directions. Under the almost certain assumption of immaterial change of carbon concentration, one can also state that volume and carbon did not change in different directions.
Extending analysis from the 50 nations in Figure 4 to all 144 nations reporting both area and volume did uncover volume and biomass changing in different directions in Israel, Liechtenstein, Mauritius and Swaziland.

The synoptic chart illuminates a peculiarity of some national values. The points representing several nations form a line along the equator of $d = 0$. These nations may have calculated the volumes $V$ reported in FRA2005 by multiplying their forest areas by the same $D$ in 1990 and 2005.

QUANDARY II: HOW ARE MEDITERRANEAN CENTRAL AMERICAN AND CARIBBEAN FORESTS FARING AFTER MILLENNIA OR CENTURIES OF EUROPEAN SETTLEMENT?

Lands surrounding the Mediterranean Sea and draining into it via the Black Sea have long been subjected to humans: Egyptians, Greeks, Persians, Phoenicians, Romans, and their successors. Romans reputedly salted the earth around Carthage, near present Tunis. The Ukraine was known, not for forests but as the breadbasket of Europe. Although these human impacts plus the Mediterranean climate have not left a landscape verdant with forests, FRA2005 can show whether after millennia, things are getting better or worse in the former Roman Empire. Table 2 summarizes the 15-year, 1990 to 2005 changes for 29 countries draining into the two Seas (The vast Russian forests, much draining elsewhere, and the Sudan are omitted.) Twenty-six of the 29 countries expanded their forests, making the change $a$ of area 0.5% annually. Density $d$ increased in all but one of the 26 countries reporting, making the change $d$ be 0.9% annually. Thus the regional volume $v$ increased 1.4% annually.

The Central American and Caribbean regions, also reported in Table 2, provide a counter point after centuries rather than millennia of European settlement. In Central America, Belize did not lose forest area, but the other six nations did. In the five that reported density, it increased in one nation, remained unchanged in two, and decreased in two. By the way, FRA2005 report of Salvadoran forests shrinking 1.5% differs from the Hecht et al. (2006) report of increasing tree cover in El Salvador.

The 24 Caribbean nations reporting forest areas were somewhat more encouraging, thanks to areal increase in Cuba, Puerto Rico, and St. Vincent. Nevertheless, most of the Caribbean nations reported no change in area. Only seven of the 25 nations reported volume. Amid falling volume in four of the seven, Cuba’s fast annual increases of 2.3% in density $d$ and 4.2%/yr in volume stand out.
QUANDARY III: WHAT VARIATION DOES THE GLOBAL SUM OF DEFORESTATION CLOAK?

Fears of climate change caused by greenhouse gases circulating around the globe focus attention on global sums. Attending only to a global sum of forests, however, obscures the lessons taught by national differences. The synoptic chart in Figure 4 and the examples above forewarn that variation is great. Within the global sum, how many nations are losing and how many gaining forests and their sequestered carbon?

FRA2005 reported the 1990 and 2005 forest area in 214 nations and volume in 144. The global sum of area fell at an annual rate of 0.21% in all 214 nations reporting area and 0.20% annually in the 144 reporting changes in volume as well as area (Table 3). The global growth of density at 0.05% annually slowed the global loss of volume to 0.15% annually and to 0.16 for biomass and carbon. The half-lives corresponding to the losses of global area, volume and biomass ranged from 333 years for area to 472 years for volume.

Rather than the global averages, however, the exercises in this section concern variation. Changes varied greatly among nations as the frequency of gain, steady and loss in Table 3 confirm. Among the 214 nations reporting 1990 and 2005 areas in FRA2005, 43% lost area. Nevertheless, among the 144 reporting volume in both years, half gained density, and about 40% gained area, volume, biomass and carbon. The gainers that were formerly losers have experienced forest transitions (Mather, Fairbairn, Needle, 1999). The global net losses of area, volume, biomass and carbon cloaked these divergent national gains and losses.

The discrepancy between global losses and frequent national gains points to great losses in a few nations. Indonesia and Brazil affected the global sum remarkably (Table 4). Indonesia, with only 3% of global growing stock V in 1990, suffered a loss fully 96% as large as the global loss during the ensuing 15 years. Brazil, with about a fifth of the total volume suffered a loss 85% as large as the global loss.
Table 3. The annual global changes and the frequencies of gains and losses. The global changes of area in 214 and 144 countries, and the frequencies of gains and losses during 1990 to 2005. The annual percentage changes are in area $a$, density $d$, volume $v$ and biomass $m$. The number shows the total nations examined. Source: FRA2005.

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</tbody>
</table>

Table 4. The volumes $V$ in 1990 and 2005 and the changes $V'$ of volume and $Q'$ of carbon. The changes are expressed as cubic meters, %/yr, and half or doubling time in all 144 nations reporting volume, and in two nations, Brazil and Indonesia. The 142 nations are sans the great losses in the two nations. The annual changes $v$ %/yr are interpreted as years for $V$ to halve (negative) or double (positive). The changing volumes are converted into carbon $Q'$ Pg/yr. The annual changes $q$ %/yr and half or doubling times for carbon equal those for volume. Source: FRA2005.

<table>
<thead>
<tr>
<th>Country</th>
<th>1990 V million m$^3$</th>
<th>2005 V million m$^3$</th>
<th>$V'$ million m$^3$</th>
<th>$v$ and $q$ %/yr</th>
<th>Years to halve or double</th>
<th>$Q'$ Pg/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>144 nations</td>
<td>391,985</td>
<td>383,434</td>
<td>-8,551</td>
<td>-0.15%</td>
<td>-471</td>
<td>-0.29</td>
</tr>
<tr>
<td>Brazil</td>
<td>88,498</td>
<td>81,239</td>
<td>-7,259</td>
<td>-0.57%</td>
<td>-121</td>
<td>-0.24</td>
</tr>
<tr>
<td>Indonesia</td>
<td>13,442</td>
<td>5,216</td>
<td>-8,226</td>
<td>-6.31%</td>
<td>-11</td>
<td>-0.27</td>
</tr>
<tr>
<td>142 nations</td>
<td>290,045</td>
<td>296,979</td>
<td>6,934</td>
<td>0.16%</td>
<td>440</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Had the single nation of Indonesia suffered no loss from 1990 to 2005, the global sum of growing stock would have remained nearly steady. And if the two nations of Indonesia and Brazil had not lost, the world would have gained growing stock at 0.16% annually, which would double volume in 440 years. Without the two nations’ losses of area, the global half-life of area would lengthen from less than 4 to more than 9 centuries.

QUANDARY IV: CAN THE IDENTITY TRANSPARENTLY AND SIMPLY ESTIMATE THE TONS OF CARBON SEQUESTERED IN FORESTS?

The development of the Forest Identity in an earlier section ended with a calculation of the $q$ % annual change of carbon. The Identity enables this transparent estimate of the percentage change of sequestered carbon that is not obscured by complex calculation. To compare the storage of carbon in forests with carbon emissions from fossil fuel, however, requires rates in absolute tons/yr. So, we now write the Identity to calculate a rate $Q'$ tons/yr rather than the state $Q$ tons or the relative change $q$ % annually. That is, a prime attached to a state, such as $Q$, signifies an annual increment in the quantity standing in a nation’s forests. Let $V'$ be the average m$^3$/yr change from 1990 to 2005 of volume and calculate $Q'$ in annual petagram (Pg) or $10^{15}$ grams or 1,000 million tons. Then

$$Q' (\text{Pg/yr}) = V' (\text{million m}^3/\text{ha/yr}) \times B (\text{tons biomass/m}^3 \text{ volume}) \times C (\text{tons carbon/ton biomass}) \times \frac{Pg}{1,000 \text{ million tons}}.$$

$Q'$ is easily approximated by assuming $B$ equals 1 and $C$ equals 0.5 (Table 4). The result is –0.29 Pg carbon lost from aboveground forest biomass. This corresponds to –0.36 Pg/yr from roots plus above ground biomass.

Alternatively $B$ may be varied among nations. In an unpublished communication Fang (2006) cited $B$ for different regions: 0.72 ton/m$^3$ for Europe and Australia, 0.74 for Japan, 0.82 for Russia, 0.94 for China, 1.01 for the U. S. A., and 1.04 for tropical

Table 5. The changes $Q'$ (Pg/yr) of sequestered carbon from 1990 to 2005 in four nations calculated with an approximation of $B = 1$ versus varied $B$.

<table>
<thead>
<tr>
<th>Country</th>
<th>$B = 1$</th>
<th>Varied $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.092</td>
<td>0.087</td>
</tr>
<tr>
<td>USA</td>
<td>0.098</td>
<td>0.099</td>
</tr>
<tr>
<td>Brazil</td>
<td>-0.242</td>
<td>-0.252</td>
</tr>
<tr>
<td>Indonesia</td>
<td>-0.274</td>
<td>-0.285</td>
</tr>
</tbody>
</table>

nations. Table 5 shows the outcome of a simple assumption that $B = 1$ and also of the alternative values in the preceding sentence. The alternative results when $B$ is 0.94 for China, 1.01 for USA and 1.04 for Brazil and Indonesia, however, differ rather little from those calculated with $B = 1$ (Table 5). The difference between the $Q'$ calculated for alternative $B$ does not encourage detailed examination beyond the four countries.

Before leaving the estimate of a -0.29 Pg yr carbon loss or emission from forests, one asks its scale. Globally during 1995 to 2004, global carbon emissions from energy consumption averaged 6.5 Pg carbon yr (U.S. Energy Information Agency, 2006). Thus the calculated loss of -0.29 carbon from forests of the 144 nations is 4.5% of the emission from energy consumption (See the right hand column of Table 4). Instead of losing carbon, the forests of the 142 nations excluding Indonesia and Brazil sequestered about 4% of global energy emissions. Even with an optimistic scenario of no loss from Indonesia and Brazil plus adding a quarter more for roots, however, forests of the 142 nations sequestered only about one twentieth of the carbon emitted from coal, oil and gas.

**QUANDARY V: DOES HEAVY HARVEST MATCH GREAT DEFORSTATION?**

The Bruntland Commission, formally called the World Commission on Environment and Development (1987), defined the unexceptionable goal of sustainable development as 1) meeting the needs of the present and 2) not compromising the ability of future generations to meet their own needs. To evaluate whether meeting present needs to harvest wood compromises future supply requires calculating the absolute m$^3$ removed annually rather the relative rate $v$. Let sustainability be a harvest of $V'$ (m$^3$/yr) from area $A$ (ha) that meets present needs without diminishing the standing inventory of $V$ m$^3$ because density growth $D'$ m$^3$/ha/yr matches harvest.

\[ V' = A \times D' \]

Note that we again signal an annual change by a prime and that $V'$ (m$^3$/yr) is not $v$ percent annually but instead equals $\left[\left(\text{a}\% /\text{yr}\right) + \left(\text{d}\% /\text{yr}\right)\right] \times V$ m$^3$. Behavior that makes $V'$ zero or positive can be called sustainable because, when future generations arrive, they will find as much or more volume as stands today.

Seeing a forest clear cut by roaring chain saws provokes the assumption that harvest affects the change $V'$ of volume, profoundly. It justifies an examination whether cutting timber for our generation compromises lumber for later generations, unsustainably. If, on the other hand, other causes dominate deforestation or growth replaces the timber cut, harvest will not affect $V'$ greatly. Figure 5 attempts but fails to find the change of volume $V'$ related to roundwood production during 2000. Roundwood encompasses industrial roundwood and fuelwood (Food and Agriculture Organization of the United Nations, 2007). As it turns out, some nations harvested far less volume than they lost, while other nations harvested much but paradoxically gained volume standing in their forests. Indonesia and Brazil harvested only a fifth to a half as much as they lost, while, China and the U. S. A. paradoxically increased the volume in their forests 1.5 to 2.5 times as much as they harvested. If some nations lose more than they harvest while other nations gain at the same time that they harvest, other forces are overwhelming the effect of harvest on deforestation, or growth replaces the harvest. Kauppi et al. (2006) concluded, “It is not forest industries themselves but rather a high density of population in combination with poverty that tends to drive deforestation”. All generalizations, however, bring amplifications and qualifications in their train (Ostrom, Nagendra, 2007; Chomitz, 2006).

**QUANDARY VI: HOW MUCH DOES HARVESTING FAST RATHER THAN SLOW GROWING FORESTS SPARE FORESTS?**

Although other forces and re-growth may overwhelm the impact of harvest on the change in volume, conservative people will nevertheless want to diminish its impact on forests, especially natural ones. We equate the impact of harvest with the forest area to grow replacement timber, sustainably. We explore how harvesting more from a faster growing forest changes impact.

Envision two forests growing at different rates. The slower grows volume at $D'_1$ (m$^3$/ha/yr) on area $A_1$ (ha), and the faster grows on area $A_2$ at $D'_2$. That is, the faster has a $D'_2/D'_1$ growth advantage. The fraction $p$ is harvested from fast growing $A_2$, while $(1-p)$ is harvested from $A_1$.

\[ V' = V'_1 + V'_2 = (1-p) \times A_1 \times D'_1 + p \times A_2 \times D'_2 \]

The impact of harvest on the slower forest, which is equated with the forest area to grow replacement timber, equals

\[ A_1 = (1-p) \times V'/D'_1 \]

Thus, increasing the proportion $p$ harvested from the warmer decreases the impact on the cooler forest, linearly.

On the broader issue of the total impact on both forests, both $D'_1$ and $D'_2$ play roles,

\[ A_1 + A_2 = [(1-p)/D'_1 + p/D'_2] \times V' \]

As $p$ increases toward its limit of 1, the total impact ($A_1 +
increase from 72.8 to 78.8%. If the area spared by the shift in harvest from the slower growing forests in the north to the fast growing forest in the south region of the U. S. A. illustrates. The (m³/ha/yr) density increases in north and south were equated with the reported net growth plus removals from the hectares of timberland (Smith et al., 2002). In the timberland in the north and south in 2002, $D'_1$ was 3.6 and $D'_2$ was 7.4 (m³/ha/yr). From 1976 to 2001, the annual harvest $V'$ from the two regions increased from 260 to 362 million m³ while the fraction $p$ from the south rose from 72.8 to 78.8%. If $p$ had remained at its 1976 value of 72.8% while the harvest increased from 260 to 362 million m³, the calculated $(A_1 + A_2)$ to replace it would have spread by 17.8 million ha. Instead, during 1976 to 2001, the fraction $p$ harvested from faster growing, southern forests rose to 78.8%. Because the greater harvest of the faster growing forest increased $p$ and decreased the area per harvested volume, the area $(A_1 + A_2)$ was 14.7 rather than the 17.8 calculated at the 1976 value of $p$. The difference between the 1976 value of $p$ and its actual, smaller 2001 value decreased area $(A_1 + A_2)$ from 17.8 to 14.7 million ha, a decrease of 3.1 million ha or 18%. In short, shifting harvest to faster growing forests spared 3.1 million ha, far more than either the 0.9 million of Yellowstone Park or 1.3 million of Connecticut.

**QUANDARY VII: HOW MUCH DOES HARVESTING PLANTATIONS SPARE NATURAL FORESTS?**

The growth in plantations versus natural forests provides even greater contrast than the warm versus cool forests. Whereas in the preceding example, growth in cool and warm forests differed only from 3.6 to 7.4 (m³/ha/yr). Brazilian eucalyptus plantations provide a contrast of 40 (m³/ha/yr) (Brazil Eucalyptus Potential Productivity (BEPP), 2003). Literature about plantations abounds in projections of the percentage $p$ of production they furnish.

With our focus on sustainability rather than production, however, we calculate how changing the portion $p$ of production from plantations changes the proportion of impact on natural forest area. If the area to replace all harvest is $A_1$, natural plus $A_1$, plantation forest, the proportion of the impact on natural equals $[A_1 / (A_1 + A_2)]$. How does this impact proportion vary with the production proportion $p$?

Begin with dimensionless $p = V'_2 / V'$, the often reported production proportion from plantations, where

$V'$ (m³/yr) = $V'$ production from natural forests + $V'_2$ production from plantations,

$D'_1$ and $D'_2$ (m³/ha/yr) = density increases in natural forests and plantations, giving plantations a growth advantage of $D'_2 / D'_1$.

$(A_1 + A_2)$ (ha) = areas of natural forests and plantations to grow, replace or match harvest $V'$, and because the areas are volumes produced divided by increases in density,

$A_1 = (1-p) \times V'/D'_1$ and $A_2 = p \times V'/D'_2$.

The reciprocal of the impact proportion that we seek is the ratio of total area $(A_1 + A_2)$ to grow all $V'$ divided by the natural area $A_1$ to grow $(1-p) V'$:

$(A_1 + A_2) / A_1 = (1 + p/(1-p) \times D'_1 / D'_2)$.

In words, the impact proportion $[A_1 / (A_1 + A_2)]$ of the replacement area or impact that is natural forest furnishing $(1-p)$ of production is the reciprocal of $1 + p/(1-p) \times D'_1 / D'_2$ of production from natural forests + $V'_2$ production from plantations.

When the production proportion $p$ is small, the impact proportion $[A_1 / (A_1 + A_2)]$ decreases nearly as $D'_1 / D'_2$; and when $p$ is large, the impact proportion decreases nearly as the plantation growth advantage $D'_2 / D'_1$.

Growth rates in Indian plantations illustrate how a greater fraction produced from plantations lowers the impact of production. Because Lal and Sing (2000) report that Indian plantations add density about twice as fast as natural forests, let $D'_2 / D'_1$ be 2 in the upper panel of Figure 6. The impact proportion falls along the curvilinear red line as $p$ increases because, as the natural area $A_1$ falls along the green line and $A_2$ rises half as rapidly along the blue line, the sum of natural plus plantation area $(A_1 + A_2)$ falls along the black line in the upper panel. At low percentages $p$ of production from plantations, a 1% step of $p$ from, say, 5 to 6%, decreases the impact proportion $[A_1 / (A_1 + A_2)]$ only from 97.4 to 96.9%, the red line. That change is the 0.5% expected with a $D'_1 / D'_2$ of 1/2. In mid-scale, a 1% step of $p$ from 50 to 51% decreases the percentage impact about 1% from 66.7 to 65.8%. When $p$ has risen to, say, 98% a step to 99% decreases the impact percentage from 3.9 to 2.0%, which is near the expected 2% when $D'_2 / D'_1$ is 2. A comparison
of cool forests versus Brazilian eucalyptus might raise the growth advantage to 8. The lower panel of Figure 6 shows how sharply a $D'_2 / D'_1$ of 8 curves the red line relating production and impact proportions.

Finally, use the same $D'_2 / D'_1$ of 2 to calculate the impact fraction of the present 33% production from plantations and of projections of 50% and 75% (Sohngen, Mendelsohn, Sedjo, 1999). In the present case of $p = 33%$ plantation production, plantations occupying only 20% of the area produce 33% of the wood, while natural forests still require fully 80% of the area to match only 67% of the wood production. If $p$ increases to the projected 50%, the area of natural forests to match production will fall from the present 80% to 67% of the total area to match production. The projected 75% production from plantations would shrink the area of matching natural forests to only 39% of the total area. As increasing $p$ decreases the impact proportion $[A_1 / (A_1 + A_2)]$, it also decreases the total area $(A_1 + A_2)$ toward $D'_1 / D'_2$ of the total before plantations.

If plantations grow twice as fast as natural forests, harvesting one hectare of plantation spares two hectares of natural forest. Harvesting timber from forests can be sustained, especially harvesting fast growing ones.

**CONCLUSION**

Diligent observers have accumulated a wealth of data about diverse characteristics of forests. By defining the valued attributes of forest area, volume, biomass and captured carbon in terms of measured variables, the Forest Identity opens the way to the quantitative analysis, logical integration and transparent summary of the wealth of data. Defining and integrating attributes in terms of measurable variables, the Forest Identity lends an importance to such statistics as reported in FRA2005 that should encourage their use, increase their value, and subsequently encourage their accurate measurement.
The logical combination of area and density as volume, the orderly relation to volume of the allometric ratio between biomass and volume, the steadiness of the ratio of tree roots to shoots, and the steadiness of carbon concentration simplify and broaden application of the Identity.

Consequently the rates of change of attributes and variables can be displayed on a synoptic chart. The graphic chart displays general patterns and reveals specific lessons within hundreds of statistics that global averages cloak. The frequency of improving forests means that deforestation accompanied by deteriorating attributes is not inevitable.

Mather, Fairbairn and Needle (1999) have documented European forest transitions from net deforestation to net reforestation. The Forest Identity quantifies other transitions in both area and volume from deforestation millennia or centuries ago to expanding forests today. The Identity furnishes a transparent estimate of carbon sequestered in forests, above and below ground, and it uncovers the paradox of the small effect of harvest on volume. It calculates how producing wood in faster growing forests lessens the impact of harvest. And, it relates the increasing production proportion of wood from plantations to the lessening impact proportion on natural forests, a relation that becomes more curvilinear as the growth advantage of plantations increases.

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